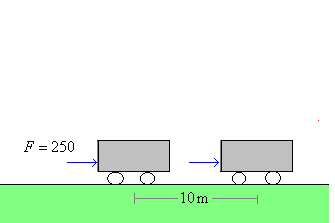
Work

**Example**

If we push a shopping cart with a horizontal force of 250N over a distance of 10m, what work will we have done on the shopping cart?

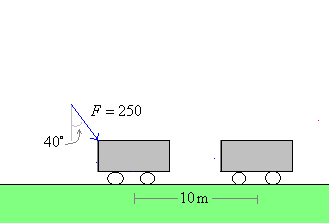


The work will be,



**Example**

If we push a shopping cart with a horizontal force of 250N at an angle of 40 degrees w/r to the vertical, over a distance of 10m, what work will we have done on the shopping cart?

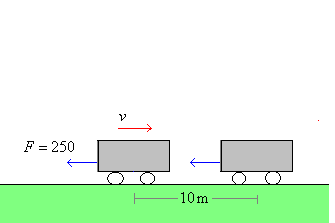


The work will be, . Now φ = 50˚ is the angle between **F** and Δ**r**. And so we have,



**Example**

Suppose the cart is rolling down the parking lot, and you’re trying to slow it down, so you exert a force in the opposite direction, while it moves through the 10m distance. What work do you do now?

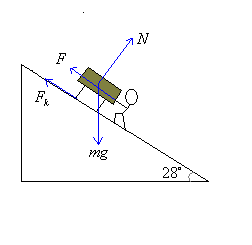


The work done is:



**Problem**

A 330kg piano slides 3.6m down a 28˚ incline and is kept from accelerating by a man who is pushing back on it parallel to the incline. The effective coefficient of kinetic friction is 0.4. Calculate (a) the force exerted by the man, (b) the work done by the man on the piano, (c) the work done by the friction force, (d) the work done by the force of gravity, (e) the work done by the normal force, (f) the net work one on the piano. Note that since the piano is sliding down the incline, the force of friction points upwards.



(a) The force exerted by the man is exerting can be determined via N2L. We add up the forces in the x direction (parallel to the plane)



and in the y-direction (perpendicular to the plane)



now plug this N into the x-equation,



(b) The work done by the man is:



(c) The work done by the friction force is:



(d) the work done by the force of gravity is:



Alternately, we could use the formula (only true for gravitational work)



(e) the work done by the normal force is:



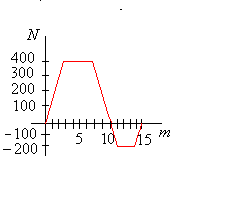
(f) finally, the net work done on the piano is:



If we had carried out our calculation more precisely, we would’ve gotten 0 for the net work. The net work is 0 because, from the Work energy equation we’ll discuss later, the block isn’t gaining any kinetic energy and so no net work is being done on it.

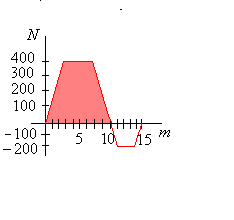
**Problem**

The force on an object, acting along the x-axis, varies as shown in the figure below. Determine the work done by this force to move the object (a) from x = 0 to x = 10m, and (b) from x = 0 to x = 15m.



The force is on the y-axis, and position is on the x-axis. To determine the work done we just determine the area under the F-x curve bewteen the initial and final positions. So for

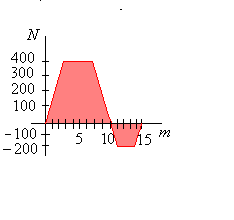
(a) We want to calculate the following area,



And it will be:



(b) to determine the work done in the second case we’d have to determine the following area,



The second area is negative (since its involving negative forces). The negative work happens when we exert a negative force (i.e. a force to the left) while the block goes to the right (from 10m to 15m). Since the force and displacement are in opposite directions during the second segment, the work done then is negative. To find the total work, we just add the two areas. So the total work would be:



Note that my diagram doesn’t quite match up with theirs, and so this would be the reason for any discrepancy.

**Example: Work done by 1D position dependent force**

Suppose a particle experiences a position dependent force of . How much work would be done on this particle moving from x = 10m to x = 3m.

Now we have an explicit formula for the force, so let’s work out the integral. The work done will be,



**Example: Work done by 2D position dependent force**

Suppose a particle experiences a position dependent force in 2D of . How much work will this force do on a particle moving between coordinates (0,1) and (3,5)?

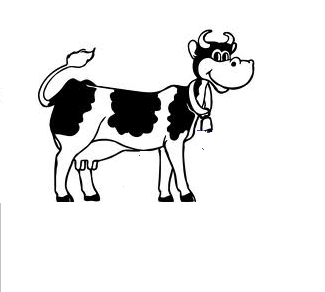
Again, we form the work function.



Energy

**Question 1 (10 points)**

A flying cow has mass m = 1850kg, and is traveling at velocity v = 13m/s at a height of 37m. What is its kinetic energy, and what is its potential energy? What is its total energy?



Power

**Example: power exerted pushing a block**

Suppose we push a block along the ground with a force of 200N, at a speed of 5m/s. What power are we exerting? Well according to the above, we’re exerting a power of



**Example: power exerted using a stair climber**

Suppose we’re on a stair climber trying to lose some weight. The machine is set to a level so that every second we step up 0.3m. If we weigh 70kg, what is our power expenditure in W? In kcal/hour?

Well, P is the rate at which we’re doing work. Every second we step up 0.5m. The work required to do this is W = FΔrcosφ = (mg)(0.3)cos(0) = 206J. If we do this every second, then, P = ΔW/Δt (to use the discrete version) is 206J/1s = 206W. To convert to kcal/hour, we use the fact that 1kcal = 4186J:



Therefore 206W = 177 kcal/hour.

**Example: calories burned on stair climber**

It is the case that the body converts stored chemical energy (obtained from food) to work with an efficiency of 20% according to Wikipedia. In that case how much energy does the body burn per unit time to generate the power output above?

Well, since the body is 20% efficient, the rate, Pin, at which it must burn energy to generate this power output, Pout, is:



and therefore



which is 886 kcal/hour. Thus in ½ hour of such exercise, you would burn 443 kcal. Using similar elementary calculations, one can show that you burn about 90 kcal per mile when running.

**Example: Hydroelectric power**

The water flows over Niagara falls at a rate of 1800 m3/s through a distance of approximately 45m. What is the rate at which gravitational potential energy is lost by the water, i.e., what is the power loss of the water? Compare this to the power requirements of a large city (P ~ 1GW)

The power is:



Now every second, an amount of water equal to:



will fall over the falls a distance of 45m. Thus the amount of potential energy lost in that second is:

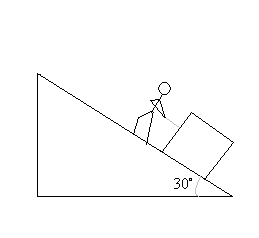


So the gravitational potential energy power loss is:



The gravitational potential energy gets converted to kinetic energy of course, as the water accelerates toward the base of the falls. But if we build a damn and hydroelectric plant, etc., we can convert the gravitational potential energy to electric potential energy instead, and thereby supply the energy needs of nearby cities. Such plants typically have an efficiency of around 20%, and so there would be ~ 160MW of power available, which is a substantial fraction of the energy needs of a large city. The way in which this can be done will be discussed in physics 2.

12. What power must you exert to pull a 75kg block up a 30˚ incline at a speed of 7m/s?



The required power is



The force required is, from N2L,



and so the power is:



Olympic sprinters accelerate from rest to approximately 11m/s in perhaps 30m. If the sprinter weighs 70kg, what power must he exert to do this?

The power would be P = ΔW/Δt, where W is the work required to get the you from a velocity of 0 to one of 11m/s in 30m. According to the work-energy equation, the work is just,



The time required is:



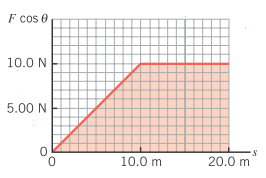
So the power is:



Work & Energy

**Problem**

A net external force is applied to a 5-kg object that is initially at rest. The net force component along the displacement of the object varies with the magnitude of the displacement as shown in the drawing. What is the speed of the object at *s* = 15 m?

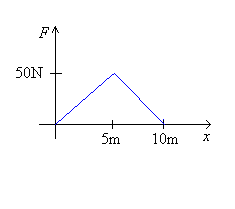


Can use the WE equation,



**Problem**

Suppose you push on a 50 kg shopping cart according to the following changing force. What will be its velocity (in m/s) after 10m?



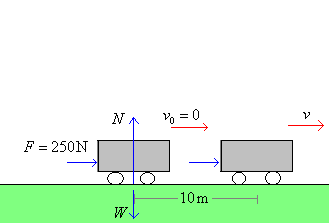
Apply the work-energy equation:



**Example: Pushing a shopping cart with a constant force**

If we push a shopping cart with a horizontal force of 250N (~ 50lbs), starting from rest, then what will be its velocity after it moves through a distance of 10m? Suppose the shopping cart has a mass of 30kg..

First we label all the forces acting on the cart. There are 3, namely F, the normal force, N, and the weight, W = mg.



All are non-conservative except for the gravitational force (there are no spring forces present). If we use the work-energy equation, then we have,



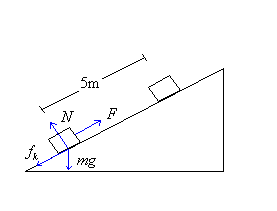
Solving for v we get,



Reviewing, we saw that N did no work because it was perpendicular to the displacement (this is always the case for N ), and only the part of a force parallel to the displacement contributes to work. Also note, the work done by the force F went entirely into increasing the carts KE. No change in PEg nor PEs occurred since there was no change in height in the first case, and no springs present anyway, in the second.

**Problem 1**

Suppose a person is pulling a 20kg sled (with the rope parallel to the surface) up an incline (angle 30 degrees) with a coefficient of kinetic friction equal to 0.2. If he pulls the sled with a force of 200N a distance of 5m, what will be the final velocity of the sled assuming it starts from rest?



Hopefully it will not be surprising that we’ll use the WE equation,



The non-conservative forces are F, fk, and N. Now N doesn’t do any work b/c it is perpendicular to the displacement. So all we have is:



Now the friction force is given by:



and the normal force is given by N2L in the y direction (y meaning perpendicular to the plane). The sum of all forces in this direction ought to be 0 since the block isn’t moving in that direction and so,



now we may continue with the WE equation,



Now Δy is given by:  and so filling in the rest of the numbers we get,



**Problem 2**

Suppose a car (m = 1500kg) and speed v = 5m/s is heading towards you. How much work would you have to do to stop it in 7s? How many Calories would you burn?

We can use the WE equation on the car.



The work on the car is the work you do, we’ll call it W. And the change in mechanical energy of the car will just be a change in kinetic energy. So we’ll have,



which would amount to:



**Problem 3**

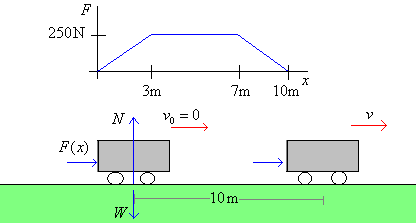
What power would you exert to stop the aforementioned car in 7s?

The power would be work/time and so,



**Example: Pushing shopping cart with a variable force**

Consider the same example as above, but this time, let our force change with position. Suppose that in the beginning it takes a little while (3m) for our force to get up to its maximum value, and then we get tired and push weaker over the final 3m.



Now we’d like to calculate the velocity of the cart by the 10m mark.



as before. Now to calculate the work done by our force, we must calculate the area under the F-x curve. This area is:



and so the final velocity will be:



4. Say you throw a 1kg basketball with a velocity of 10m/s to the right. As soon as it crosses the *x* = 0 mark, it experiences a position-dependent force of . At what position, *x*, will it come to rest?

Use the work-energy equation:



**Question 1**

Suppose you throw a 1kg basketball with a velocity of 10m/s to the right. As soon as it crosses the *x* = 0 mark, it experiences a position-dependent force of . At what position, *x*, will it come to rest?

We can use the work-energy equation,



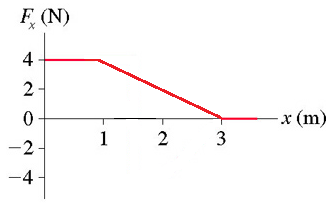
Let be d be the x-coordinate where it stops. Then the work done on it will be:



and this equal to the change in energy, which will just take the form of kinetic energy. So we have,



**Question 9**. A 0.220kg block is pushed along a table (with force F graphed below) with coefficient of friction μk = 0.15. How fast is it moving after 3m?

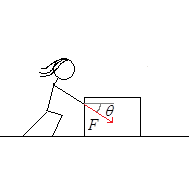


Using the work-energy equation we got:



**Problem 1.**

Suppose Cinderella pushes a 45 kg box along a frictionless floor as shown below, with a force F = 200N directed at an angle θ = 29◦. If she pushes the box through a distance of 24m, what will be its speed at that point?

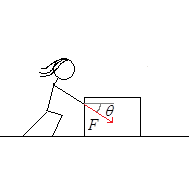


We have:



**Problem 2.**

Suppose Cinderella pushes a 45 kg box along a frictionless floor as shown below, with a force F = 200N directed at an angle θ = 29◦. If she pushes the box through a distance of 24m, what will be its speed at that point? Let there be a kinetic friction coefficient μk = 0.17



Again,



Now to get FN we use N2L in y-direction:

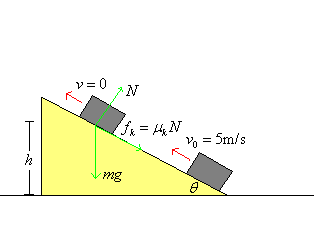


Filling this into WE equation:



**Example: Shoving box up an inclined plane w/ friction (work done by g, fk)**

Suppose we shove a 40kg box up an inclined plane at a velocity of 5m/s. If μk = 0.25, and the angle of inclination is 20 deg, to what height will the block go before coming to rest?



We will apply the work-energy equation. Let Δr be the distance the block travels up the incline.



Take a minute to observe again that the work done by normal force is 0 since it is perpendicular to the motion of the block. Now from N2L,



So we have,



and also recognize that h = Δrsinθ, and so Δr = h/sinθ. So,



Therefore,



Note how the answer didn’t depend on the mass. Now let’s go back and measure the work done by each force:





and,



**Example: Throwing ball up in the air**

How high will a ball go if you throw it upwards with a velocity of 25m/s? We already know how to do this with N2L, but we’ll do it this way too. Apply the W-E equation to the ball,

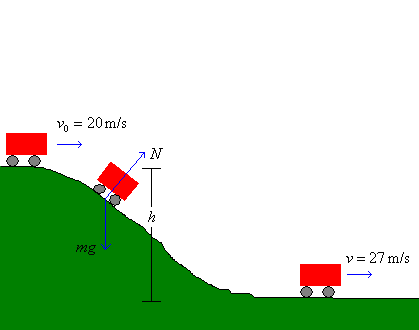


The only force acting on the ball while it is in flight is gravity – which is a conservative force. So we don’t have any non-conservative forces acting on the ball. Therefore we have,



**Example: Measuring the height of a hill or roller coaster**

Suppose you’re driving along a road, perhaps 8th avenue, and you want to measure the height of the hill there. You can do this in your car in the following way. Put the car in neutral at the top. Suppose you’re going 20m/s. And then let gravity pull the car down the hill. Then note your speed at the bottom, say 27m/s.



Then we can get h by applying the work-energy equation.



Before continuing, note that the normal force is always perpendicular to the motion of the object since N is perpendicular to the surface and the motion is parallel to the surface. Therefore N never does any work. So we have,



So note that we do not need to know the mass of the car, for example.

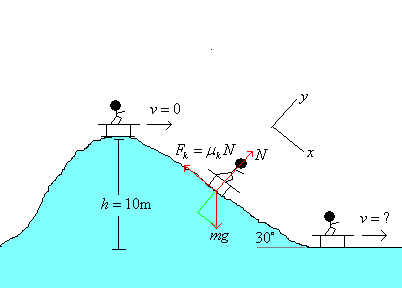
**Question 3**. Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling friction as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at 10.0° and the coefficient of rolling friction is 0.40. What length of a ramp will stop a 15,000 kg truck that enters the ramp at 28m/s?

We must use the work-energy equation:



**Example: Skating down a hill with friction**

Suppose you ride a sled down a hill 10m tall with a 30 degree incline. Suppose the coefficient of kinetic friction between the sled and snow of 0.2. And suppose your mass, along with the sled is 75kg. What will be your speed at the bottom?



We can use the W-E equation.



This time we have a non-conservative force present – friction. The friction force is, from N2L,



and the work done by this force is:

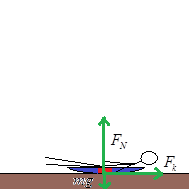


Therefore,



2. Suppose you see Captain America sliding on his shield down the freeway. If the coefficient of kinetic friction between his shield and the road is μk = 0.15, and he has an initial velocity v0 = 28m/s, how far will he slide before coming to rest?

Work-energy equation. Surprise! Let’s take the point of view that the Captain is sliding to the left, for the sake of discussion.



Then we’ll have:



Now from N2L in y-direction we have:

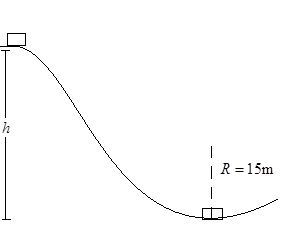


Plugging this into our WE equation we get:



**Problem 7.**

A roller coaster manufacturer wishes to build a coaster that will simulate twice the acceleration due to gravity. From what height should a roller coaster be released from rest so that its centripetal acceleration at the bottom of the curve will be equal to 2g? Note the radius of curvature of the curve is given to be 15m.



The requisite speed at bottom is:

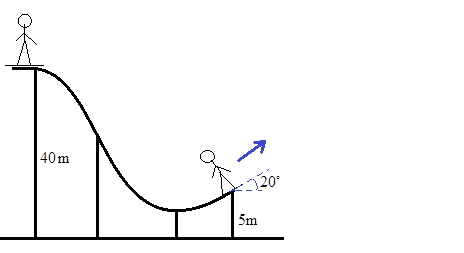


And now required speed at top to make this happen is given by WE equation:



So roller-coaster must start 14.9m above the ground.

3a. Suppose you are skiing down the frictionless slope shown below. Use the work-energy equation to determine your speed as you leave the ramp. You may assume that your speed at the top is zero.



From WE we have:



3b. Use the projectile motion stuff to determine how far away from the ramp you will land.

Your position and velocity equations will be:



and,



You will hit the ground when,



Your position at that time will be:



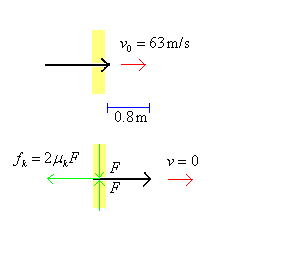
3c. Determine the maximum height you will reach along the trajectory too.

To get your max height, we can use either WE or projectile motion stuff. Let’s use WE. So we have:



**Example: Catching an arrow**

You might remember from the Scorpion King, that the evil guy catches arrows with his bare hands. Let’s examine whether or not this is possible. We’ll suppose that the bow is fired as above, with a velocity of 63m/s.

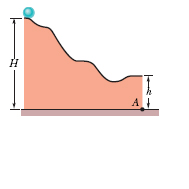


Then we use our hands (the yellow rectangles) to exert two forces, F, on the sides of the arrow. This will result in two friction forces μkF (one for each side of the arrow) which will slow the arrow down. Now it must be slowed to rest in a distance of the length of the arrow (0.8m). To estimate μk we would estimate the angle at which the arrow would slide down our hands (perhaps 30 degrees) – so μk = tan(20) = 0.36. Now we can use the work-energy equation to calculate the force, F, required to stop the arrow,



which is quite reasonable. If you can exert 76 lbs. of force with each hand (equivalent to bench pressing 152 lbs perhaps) then you could stop the arrow, provided you time things correctly. Interesting!

1. In the figure, a solid 0.2 kg *disk* rolls smoothly from rest (starting at height *H* = 10 m) until it leaves the horizontal section at the end of the track, at height *h* = 2 m. How far horizontally from point *A* does the ball hit the floor?



Speed at bottom is given by work-energy equation. Since there is no net work done on the disk, energy is conserved and we have:



And now we can use a kinematics equation to get time of flight.

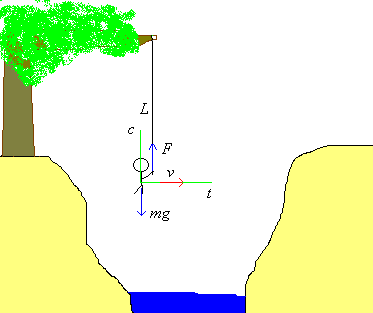


and so the distance traveled is:



**Example: Swinging on a vine**

Suppose you’re swinging on a vine over a crocodile-infested river. Suppose L = 10m and θ is 30 degrees. Also that you have a mass of 70kg. If you swing from the vine, what maximum force will you have to exert in order to hold onto it?



This is a case of circular motion, since the person is going to swing in a circle. The maximum force that you’ll have to exert will occur when you’re at the bottom of the swing – as will become apparent when we do the analysis. So show the person there, with all the forces acting on him, and the tangential, centripetal axes clearly labelled. According to N2L then, we have,



We can calculate F therefore, if we know the velocity, v. This we can get from energy conservation. Use the work energy equation between the initial position and the final position – when he is directly underneath the rope.



Now the only non-conservative force acting on the person is the force with which he pulls on the rope. But this does no work, since it is always directed along the rope, and therefore perpendicular to his velocity (and therefore d**r**). So there is no non-conservative work being done.



in this step we’re treating the top of the tree branch as y = 0. So finally,



and filling this in we have,



Its interesting to note that the answer doesn’t depend on the length of the rope. So filling in the values we get,



So the force would be over twice the person’s weight,

2. Suppose you have a mass of 60kg. If you swing on a vine from a height of 5m, what will be the tension in the vine at the time you reach the bottom?

The tension will be:



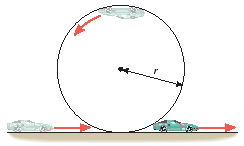
and v will be given by



So filling this in we get,



5. The drawing shows a version of the loop-the-loop trick for a small car. If the car is given an initial speed of 10 m/s, what is the largest value that the radius *r* can have if the car is to remain in contact with the circular track at all times?



Using WE equation to relate the speed of the car at the bottom to its speed at the top we have:



Now use N2L to determine what minimum speed at top must be in order to not fall off.

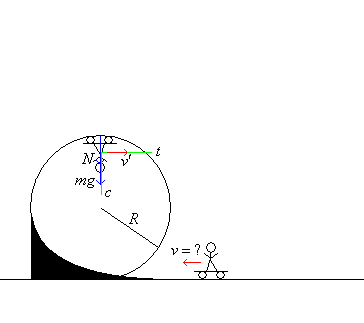


Now we plug this relationship into the first equation:



**Example: How fast need go in order to complete loop-the-loop skateboard thing.**

Suppose you’re riding a skateboard through a loop-the-loop. How fast do you need to go in order to not fall off the track?



This is another case of circular motion. The place where the skateboarder would have the highest propensity to fall would be at the top. So if we ensure that he not fall off there, he will be safe everywhere. As before now, we orient our t and c axes to point along the velocity, and towards the center of rotation. Then write out N2L



Now when a person falls off the track, they are losing contact with the track. If this happens then N = 0, since the track will no longer be pushing on them. To find the minimum speed v, necessary to complete the loop, we will consider that he is just barely in contact with the track at the top, and so we will set N = 0 (or really, just a hair above 0).

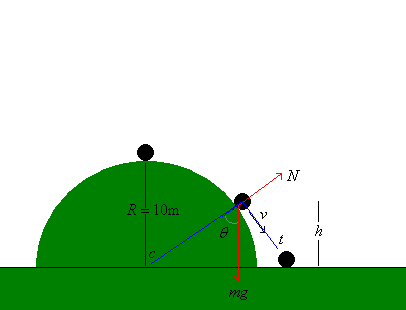


Now in order for v′ to be √(gR) at the top, we must have through energy conservation (since the normal force N does no work on the skateboarder),



**Example. Skating on hill – when fall off?**

Consider you start at the top of a semi-circular hill and roll down with your skateboard (we’ll approximate you as a ball). If the hill is 10m high, at what height will you fall off the hill, i.e., when will you lose contact with the hill?



You will lose contact with the hill when the normal force is equal to 0, by definition. So we’ll apply Newton’s second law, and see at what height, h, N will equal 0. Now Fg = mg has components in both the tangential and centripetal directions.



Now when N = 0 we have, (the tangential component isn’t important for our considerations)



Next observe that cosθ = h/R. You can see this by examining the triangle formed by the centripetal axis, the height h, and the horizontal line connecting them. Therefore,



All we need to know now is what v2 is. This can be obtained from the WE equation. Let our initial point be at the top of the hill and the final point be where we fall off. Then,



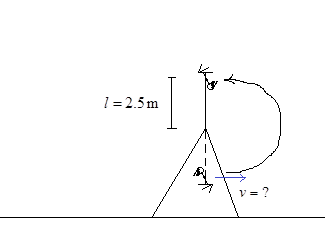
Therefore, plugging this in…



Note how the height does not depend on the gravity – interestingly enough. Nor does it depend on your mass. Therefore, every skateboarder would fall off at the same point, and also regardless if you were on the moon, or on Earth.

**Problem 4**

Wishing to apply physics to your baby-sitting job, suppose you want to push a child over the top of a swing like this. Let the length of the swing be ℓ = 2.5m. What initial velocity must you give the child to accomplish this? Note that you don’t want to kill the child so that means she must reach the top with enough speed to keep going around the circle without the rope becoming slack and collapsing. That is to say, you want her to reach the top with speed large enough that FT (tension in the rope) is just barely 0.



First we must determine the necessary speed at the top. From N2L in y-direction this is:



And now use WE equation to determine how fast she must go at bottom to reach top with that speed.

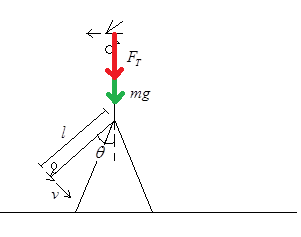


4. Suppose Tarzan is swinging from a vine 5m long. If at the bottom of the swing he has a speed of v = 7m/s, what is the tension in the vine? Assume Tarzan has a mass of 70kg.

According to N2L in the centripetal direction we’ll have,



5. A child is given an initial velocity v, at an initial angle θ = 38°. What must this velocity v be *at least*, in order that the child makes it all the way around to the top without falling off if the length of the swing is ℓ =3.5m? Note that the child’s velocity is not zero at the top; and you will need to use both the Work-Energy equation, as well as N2L for circular motion.



Let us apply the WE equation to the child, between her present position and the top. Taking y = 0 to be at the axis of rotation, her initial height is y0 = -ℓcosθ. Her final height is y = ℓ. Her initial velocity is v and her final velocity is v´, whatever velocity she must have at the top of the swing. Filling these into the WE equation we have:



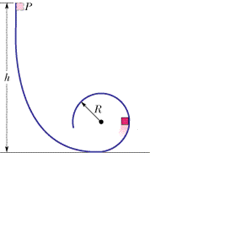
and N2L for circular motion say that:



Plugging this into the WE equation we get:



2. In the figure, a small block of mass *m* = 1 kg can slide along the frictionless loop-the-loop, with loop radius *R* = 2m. At what height *h* should the block be released from rest so that it is on the verge of losing contact with the track at the top of the loop?



If on verge of losing contact with top, then FN = 0, and moreover according to N2L at top:



Now using the WE equation between starting point and top of loop we have:



Filling in previous resutl for v = (gr)1/2 gives us:



**Example: Calculating amplitude of spring oscillation**

Suppose we take a spring (k = 10 N/m), and place a mass (m = 2kg) at the equilibrium position. Suppose then we kick the mass giving it a velocity of 5m/s. Then what will be the amplitude of its oscillatory motion?

Using the work-energy equation



so,



**Example: Calculating A from arbitrary position**

Consider the same apparatus. Suppose we find the velocity of the mass to be 4m/s at the position x = 3.7. What is the amplitude of motion and what is the maximum velocity of the spring?

Energy conservation implies,



Therefore, to find the amplitude we equate,



and

the max velocity can be obtained similarly,



**Example: Pole vault guy**

Examine how high one may possibly pole vault.

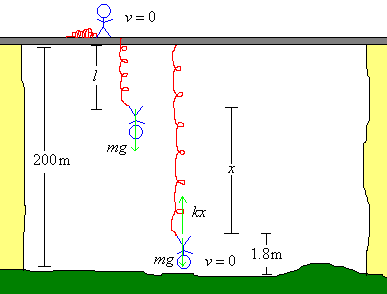
3. Suppose you have a bow and arrow w/ spring constant 1500N/m. If you pull the arrow back 0.85m below the bow and release it, how high above the bow will it go? Assume the mass of the arrow is 0.125kg.

Let d be the distance it was pulled back, and h the height it attains at top of trajectory. Then using work-energy equation…



**Example: Bungee cords**

Suppose that you want to jump off of a bridge 200m high bridge with a bungee cord tied to your waist. You want the cord length to be such that you come to rest just above the ground. If the spring constant of the cord is k = 500N/m, how long should you make the cord? Suppose you’re 1.8m tall, and your mass is 70kg. What will be your period of oscillation? What will be your amplitude of oscillation about equilibrium point? What will be your max speed as you oscillate around the equilibrium point?



Let ℓ be the length of the cord, and x the amount it stretches by the time we come to the ground. We want ℓ + x + 1.8 = 200m. Now apply the work-energy equation, letting the top be our initial position and the bottom be our final position,



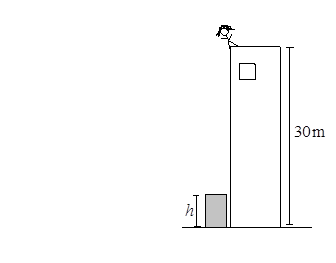
again, no non-conservative forces act on us. So Wnc is 0. Now our initial height is 0, and our final is –200.. Initially, the spring/bungee cord is unstretched, so x = 0, in the final case it is stretched a distance x. And our velocity is 0 in both cases. So we have,



Therefore we must have ℓ = 200 – x – 1.8 = 177.2m. Period is T = 2π√m/k = 2π(70/500)^(0.5) = 2.34s.

Position about equilibrium point is x(t) = xeq + Acos(ωt + φ0). xeq. = ℓ + mg/k = 177 + (70)(9.8)/(500) = 178.4m. Amplitude is therefore A = 200 – 178.4 = 21.6m. And so max velocity will be v = Aω = (21.6m)(√k/m) = (21.6)√(500/70) = 57.7 m/s.

4. Rapunzel (m = 50kg) escapes from her cell, runs to the top of the prison tower and steps off from an initial height of 30m. Before she does you stack a bunch of pillow cushions with spring constant k = 2500 N/m to a height *h*. What must this height *h* be so that Rapunzel lands on the ground with velocity v = 0? How long will it take for her to compress the spring to the ground? Note you need to use the WE equation here, and will be treating the cushions as a spring.



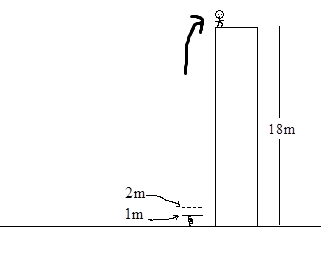
WE equation. Yawn….note that the two forces acting on her as she falls towards the ground are gravity, and eventually the spring force supplied by the cushions. Each of these is conservative so ΣWn.c. = 0. Also, her initial and final velocities will be zero. Her initial spring potential energy is zero b/c the cushions aren’t compressed initially. When she hits the ground, the cushions are compressed a distance h, and so the spring potential energy will be (1/2)kh2. Finally, her initial y position is 30, and final is 0. Putting this all in the WE equation we have:



The time it would take to compress the spring is just t = T/4. Now T = 2π√(m/k) = 2π√(50/2500) = 0.88s.

**Problem**

Suppose you construct a spring-loaded contraption to propel you to the top of a building 18m high. Your mass is m = 65kg, and you plan on compressing the spring 1m from its equilibrium position at 2m so that you will start 1m off the ground. What must the spring constant of the spring be?



We have:



**Question 3**

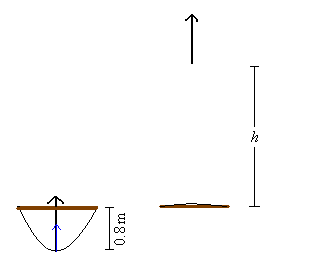
Suppose you put a 20g mass on a vertical spring (k = 500N/m). If you compress the spring 10cm and let it go, to what height (above its original uncompressed position) will the mass rise?

Use the work energy equation again. Let the initial point be when the object is compressed 10cm, and the final point when the object is at its highest spot. Then we have,



**Example: Firing arrow into the air**

Suppose you have a bow and whose cord has an effective spring constant k = 625N/m, like in the previous example. If you pull it back 0.8m, how high will it go? Assume the mass of the arrow is 100g again.



We can use the W-E equation,



the forces acting on the arrow are only the spring force, and gravity. There are no non-conservative forces acting on the arrow therefore. So we have,

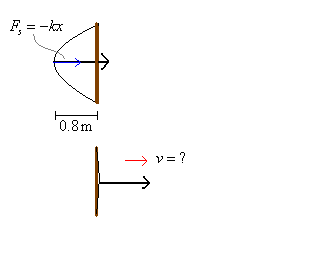


The initial velocity and final velocity are both 0 (since it starts at rest, and it also comes to rest at the highest point). The change in height is h+0.8, and the initial x position 0.8, while the final is 0. Therefore we have,



**Example: Speed of an arrow**

Using our knowledge we can estimate the speed of an arrow, being shot from a bow. The estimate turns out to be pretty good. Suppose we have an arrow, which weighs approximately 100g. Then we put it on a bow, and pull the string back. If we exert a force of approximately 100lbs ~ 500N to pull arrow/bowstring back 0.8m, what will be the velocity of the arrow when fired?



Well, first of all, if it takes a force of 500N to stretch the bow-string 0.8m, then it has a spring constant of:



Then to determine the speed of the arrow, we can use the work energy equation again,



**Problem 5**

Suppose you have an arrow with mass 50g, and a bow-and-arrow with a bow string with an effective spring constant k = 500N/m. If you put the arrow on the bow-string, draw back 75cm from the bow, and shoot the arrow horizontally from a height of 1.5m, how far away will it land?

First use WE to get the speed of the arrow,



and now the distance it will land, away is:



t is given by:



plugging into top equation,



**Question 4**. A mass m = 0.80kg slides along a table with kinetic friction coefficient 0.25 when suddenly it hits a horizontal spring (k = 40 N/m) with an intial speed v = 4 m/s.  How far will it compress the spring before coming to rest?

Again, using Work-energy equation:

